Strip Line Hybrid Junction*

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Summary—**The equivalent circuit of a strip line network is shown to display the properties of a hybrid junction. An application is illustrated by design of a balanced mixer and the presentation of the resultant measured data.**

INTRODUCTION

IN APPLICATIONS where two signal sources supply a common load, a high degree of isolation is required between the two sources if impedance matching between the load and one of the sources is not to be affected by the output impedance of the second source. This requirement is readily met through the use of a hybrid circuit. Hybrid circuits are of two general types.[1]

1) Hybrid rings or "rat races," where the isolation is accomplished by having two equal coupling paths between the two sources. At the desired operating frequency these path lengths differ by one half wavelength.
2) Hybrid junctions or magic tees, wherein isolation results from the symmetry relations between the electric and magnetic fields excited by the first input and those excited by the second input.

In the hybrid ring the isolation is frequency dependent; however, in the hybrid junction the isolation is independent of frequency. An additional feature of such a junction is that it incorporates independent tuning adjustments for matching the two sources to the load. Consequently a hybrid junction is preferable for wideband operation. For this reason a strip line hybrid junction was developed.

This report describes the action of one particular type of strip line hybrid junction, together with the design aspects and experimental results obtained in a typical application.

GENERAL HYBRID JUNCTION

An equivalent circuit for the hybrid junction is given in Fig. 1(a), which is readily converted to the representation in Fig. 1(b). It is apparent that with respect to source 3 the loads R_1 and R_2 appear in parallel, while for source 4 the loads appear in series. At the load junction the voltage developed by source 3 is unbalanced, but the voltage there resulting from source 4 must be balanced with respect to ground.

In strip line where the dominant mode is TEM, the primary problem in constructing a magic tee is the design of a frequency insensitive balun for transforming the normally unbalanced transmission line to a line balanced with respect to ground.

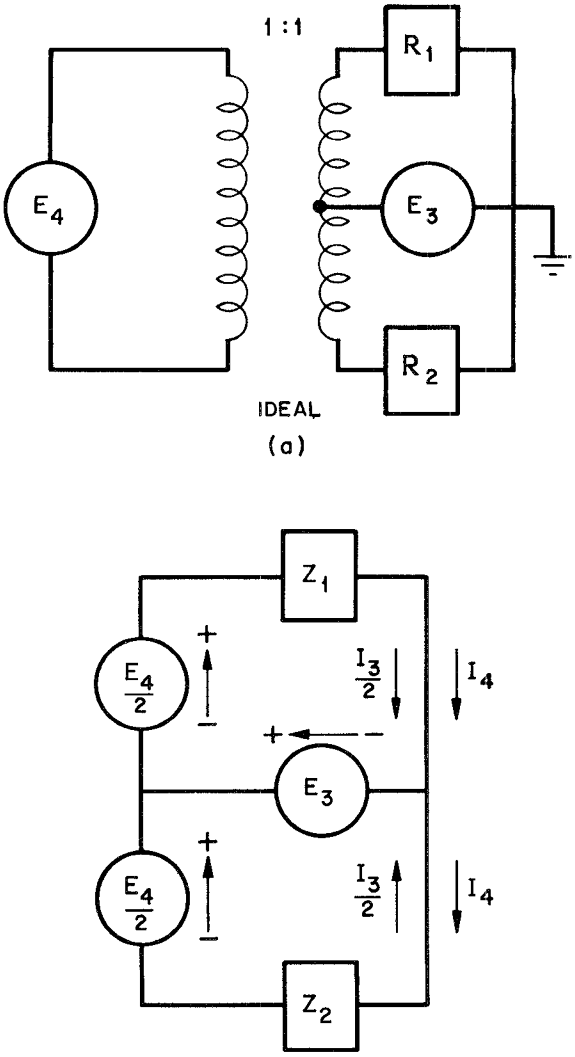

Fig. 1—Equivalent circuit of magic tee junction.

WIDEBAND BALUN

The balun to be considered[2] here may be represented as in Fig. 2(a). The individual conductors a, b, c are planes parallel to w, the ground plane. The equivalent

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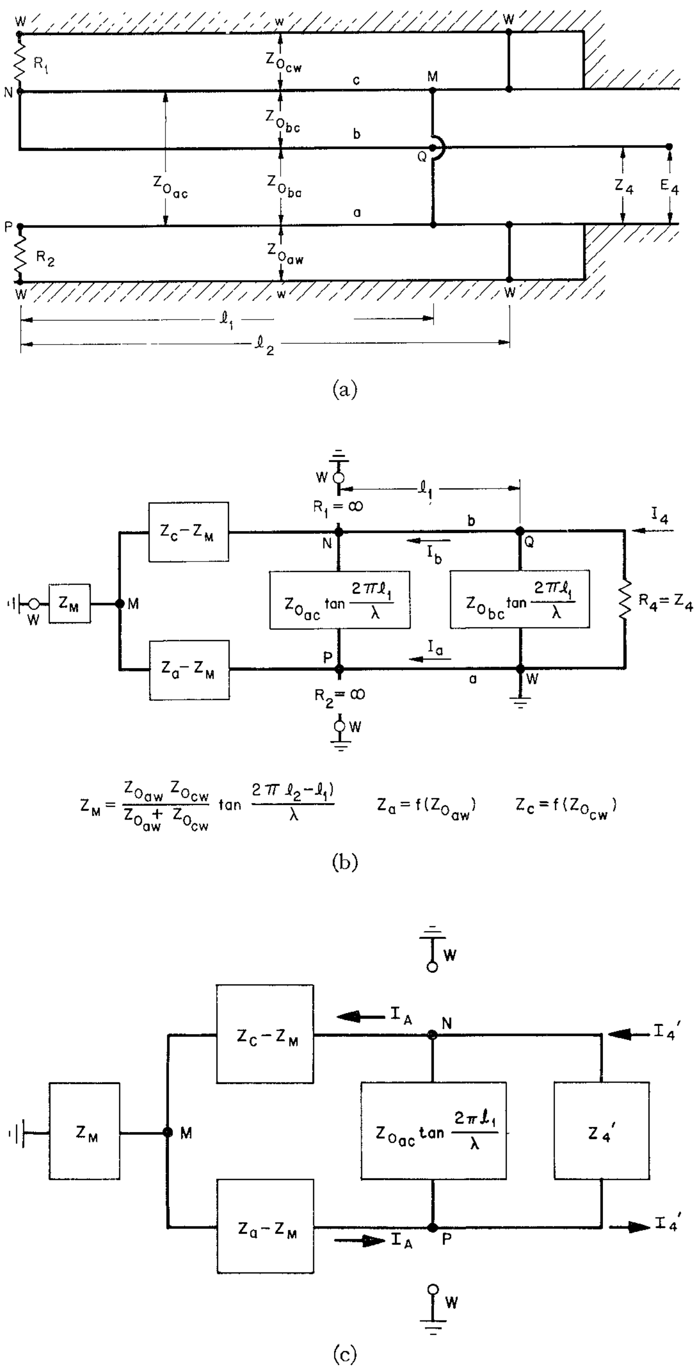
[1] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 8, ch. 9; 1948.

[2] E. G. Fubini and P. J. Sutro, "A wide-band transformer from an unbalanced to a balanced line," Proc. IRE, vol. 35, pp. 1153–1155; October, 1947.



(a) (b) (c)

Fig. 2—(a) Wideband balun. (b) Equivalent circuit of balun. (c) Modified equivalent circuit.

circuit of this balun, for the unloaded condition at the output, is given in Fig. 2(b). Where the transmission line conductors b and a are planes, the transmission line currents I_a and I_b are restricted to the inside surfaces of a and b. The unbalanced input line of characteristic impedance Z_4 and open circuit voltage E_4 is replaced with the equivalent current source.

It is convenient to designate the unbalaneed currents and voltages as symmetrical mode components I_s and V_s respectively. Likewise the balanced currents and voltages are designated as antisymmetrical current and voltage I_A and V_A respectively. In terms of the conductor currents and voltages at the output the definitions are:

$$V_s = \frac{V_{NW} + V_{PW}}{2} \tag{1}$$

$$I_s = \frac{I_b + I_a}{2} \tag{2}$$

$$V_A = \frac{V_{NW} - V_{PW}}{2} = \frac{V_{NP}}{2} \tag{3}$$

$$I_A = \frac{I_b - I_a}{4} \tag{4}$$

where V_{NW} and V_{PW} are the voltages of the nodes N and P with respect to the ground node W, and I_b and I_a are the conductor currents of the transmission line $b-a$ entering the nodes N and P.

The unbalance of the output voltage and current may be expressed by the ratios of the symmetrical to the antisymmetrical components. The criteria for perfect balun function is that these ratios both be equal to zero.

$$\frac{V_s}{V_a} = 0 \qquad \frac{I_s}{I_A} = 0. \tag{5}$$

If the output is short circuited at the node pair NP then the values of the various components are:

$$\begin{aligned} V_s &= 0 & I_s &= 0 \\ V_A &= 0 & I_A &= I_b = -I_a. \end{aligned} \tag{6}$$

For the short-circuited output both components of voltage are zero and the output current is completely antisymmetrical. Thus the transmission line to the right of the node pair NP may be replaced with a generator whose short-circuit current is completely antisymmetrical. In the modified equivalent circuit of Fig. 2(c) Z_4 and I_4 are the generator impedance and short circuit current referred to the plane of node pair NP. In the open-circuit condition the balun output is seen to be a reactive network excited by an antisymmetrical current generator. From (1) and (5) the open-circuit condition to be satisfied is:

$$V_s = \frac{V_{NW} + V_{PW}}{2} = 0. \tag{7}$$

If one considers the terminal pair mw open-circuited this becomes

$$V_s = I_A(Z_c - Z_a) = 0. \tag{8}$$

This requirement is fulfilled by setting:

$$Z_{0_{cw}} = Z_{0_{aw}} \tag{9}$$

then for any arbitrary value of Z_M

$$V_{PW} = -V_{NW}. \tag{10}$$

Thus, although the magnitudes of the antisymmetrical or balanced short-circuit current and open-circuit voltage are functions of frequency, the corresponding values of symmetrical or unbalanced components are zero at all frequencies.

The unloaded network has been shown to have perfect balun function independently of frequency; for this to remain true in the loaded condition requires the equality of the loads, *i.e.*,

$$R_1 = R_2. \tag{11}$$

STRIP LINE HYBRID JUNCTION

A strip line hybrid junction is shown in Fig. 3. It consists of the above described balun to which is joined a symmetrical mode input network formed by the conducting planes *d* and *e* together with the ground plane *w*.

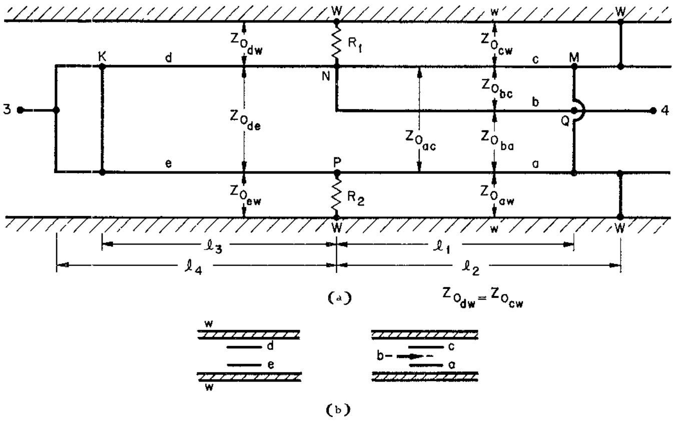


Fig. 3—Strip line magic tee junction. (a) Transmission line diagram. (b) Cross sections at plane *n-p-w*.

The equivalent circuit of the symmetrical mode input network is given in Fig. 4. In this circuit the reactance of the shorted line $Z_{0_{de}}$ connected between the nodes *N* and *P* can be excited by the antisymmetrical mode. Antisymmetrical currents on conductors *d* and *e* cancel at node *K* when

$$Z_{0_{dw}} = Z_{0_{ew}}. \tag{12}$$

Observing this condition together with the equality of the loads R_1 and R_2 insures that symmetrical input at terminal pair 3 produces equal voltages across the node pairs *NW* and *PW*.

The equivalent circuit of the hybrid junction is given in Fig. 5.

The midband wavelengths for inputs 3 and 4 are designated as λ_3 and λ_4 respectively. The conditions for unity power transfer (*i.e.*, zero reflection) at the center frequencies are:

Symmetrical network 3

$$l_2 = l_4 = \lambda_{3/4}$$

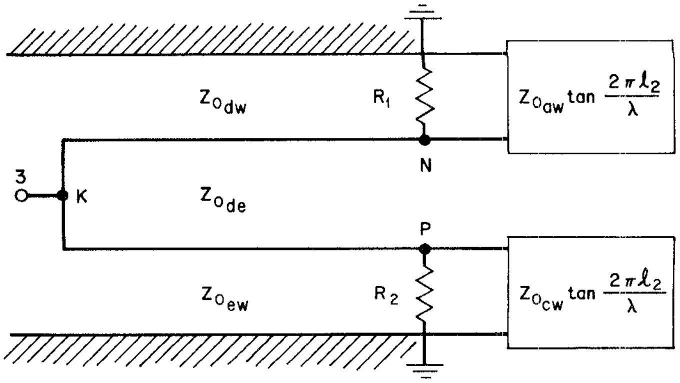


Fig. 4—Equivalent circuit of input 3.

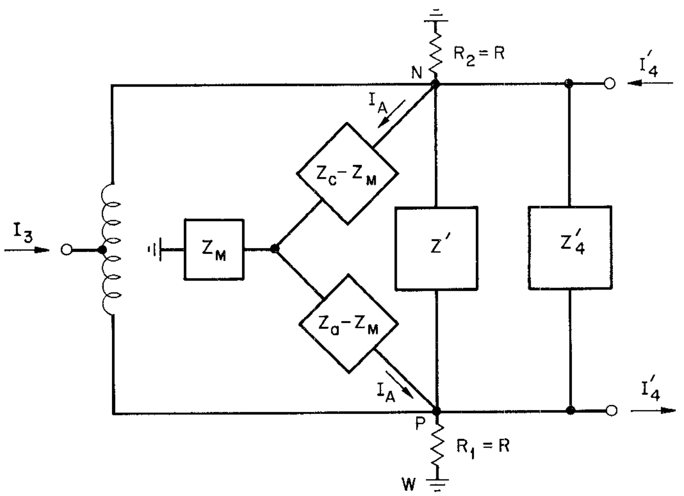


Fig. 5—Equivalent circuit for hybrid junction.

therefore

$$(Y_{0_{aw}} = Y_{0_{cw}}) \cot \frac{2\pi l_2}{\lambda_3} = 0$$

$$Z_{0_{dw}} = Z_{0_{ew}} = \sqrt{2R \cdot Z_3}.$$

Antisymmetrical network 4

$$l_1 = l_3 = \frac{\lambda_4}{4}$$

therefore

$$(Y_{0_{ac}} + Y_{0_{de}}) \cot \frac{2\pi l_1}{\lambda_4} = 0$$

$$Z_{0_{ba}} = \sqrt{2RZ_4}.$$

Assume the two center frequencies approximately equal and circuit *Q*'s low.

Then at a frequency between the two center frequencies the inputs 3 and 4 are each closely matched to the loads R_1 and R_2. Input 3 excites the loads symmetrically, input 4 excites them antisymmetrically. The elements of the scattering matrix are thus:

$$S_{33} = 0 \qquad S_{44} = 0 \qquad S_{34} = 0 \quad S_{43} = 0.$$

$$S_{31} = S_{13} = \frac{1}{\sqrt{2}} \quad S_{41} = S_{14} = \frac{1}{\sqrt{2}}$$

$$S_{32} = S_{23} = \frac{1}{\sqrt{2}} \quad S_{42} = S_{24} = -\frac{1}{\sqrt{2}}$$

Hence:

$$S_{11} = 0$$
$$S_{22} = 0$$
$$S_{12} = S_{21} = 0.$$

The scattering matrix thus has the form

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix}.$$

Because of the symmetrical nature of the junction the power division between R_1 and R_2, when fed from either 3 or 4 input, remains equal for all frequencies. The relative magnitude of symmetrical and antisymmetrical mode energy developed in the loads at a given frequency depends upon the relative value of the two transfer functions at the frequency in question.

In addition to the properties of the junction discussed above, which correspond to those commonly associated with hybrid junctions, there is the property of independent impedance matching of the loads to the individual inputs 3 and 4. This feature is particularly useful when the loads are not purely resistive or when two generators of widely different frequencies are to be employed. The length l_2 can be adjusted for matching the loads to the 3 input while l_1 and l_3 may likewise be adjusted for optimizing the match of the loads to the generator at input 4. There is no interaction between these two sets of adjustments. The only restrictions are that $l_2 \geqq l_1$ and $l_3 \leqq l_4$. Therefore it is apparent that 3 should be the low frequency input.

Design of a Balanced Mixer Employing a Strip Line Magic Tee

Initially a strip line magic tee junction was constructed following exactly the geometry as shown in Fig. 3(a) and 3(b). Since a balanced mixer was one of the more immediate applications contemplated, the line impedances were selected so as to provide impedance matching between fifty ohm input loads and two approximately equal crystal diode impedances. In the first model constructed, a pair of equal resistances equivalent to the crystal resistance was used to simulate the crystal diode loads. With this model, designed to operate in the uhf region, it was found possible to tune the inputs independently over a two-to-one frequency range, and throughout this region to maintain an isolation

$$\left(\frac{\text{power in 4 or 3}}{\text{power out 3 or 4}}\right)$$

between inputs in excess of 30 db.

On the basis of these results a new model was designed with the intention of achieving a reasonably compact and convenient construction. The only additional transmission circuitry required for the mixer is a band reject filter at the output of the IF terminals of the crystals, to provide a very low impedance of the rf return path to ground over a large frequency range. The location and construction of the filter are indicated in Fig. 6. The principal change in geometry was that instead of the transmission lines from 3 and 4 to the *NPW* plane lying on the same axis they were brought parallel to each other for reasons of space reduction and convenience of construction. They were, however, arranged so that symmetry with respect to the ground planes was preserved. To avoid mutual coupling the spacing between the axes of the lines was made much greater than the spacing between the conductors of either line. In Fig. 3(a) and 3(b) all the spaces between the conductors are assumed to have a relative dielectric constant of $\epsilon_r = 1$. However in the actual construction, the space between the planes containing conductors *a* and *c* as well as between conductors *d* and *e* contains teflon-fiberglass with $\epsilon_r = 2.5$. Aside from these differences in construction the arrangement and connection of conductors in the second model was the same as shown in Fig. 3(a).

Impedance measurements indicate that, in the uhf region, silicon and germanium microwave crystal diodes can be approximated as a parallel *RC* circuit with $R = 220$ ohms, $C = 3.75$ $\mu\mu fd$. With the load impedances thus determined the values of $Z_{0_{ba}}$, $Z_{0_{dw}}$, and $Z_{0_{ew}}$ are fixed for the proper impedance transformation to the 50 ohm inputs. Thus, with crystal diode loads,

$$Z_{0_{dw}} = Z_{0_{ew}} = 2\sqrt{110 \cdot 50} = 148.6\Omega$$

for a length $(\lambda_3)/4$ starting at the *NPW* plane in the direction of input 3. For these lines $\epsilon_r = 1$. Since $f_3 = 390$ mc, $\lambda_3/4 = 7.58$ inches. The balun output is terminated in a resistance of 440 Ω at the frequency f_4. To obtain the required 8.8:1 transformation ratio within reasonable limits over a sufficiently wide frequency band a cascade of three one-quarter-wave length transformers was employed.[3] Starting at the *NPW* plane the characteristic

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[3] J. C. Slater, "Microwave Transmission," McGraw-Hill Book Co., Inc., New York; N. Y.; 1942.

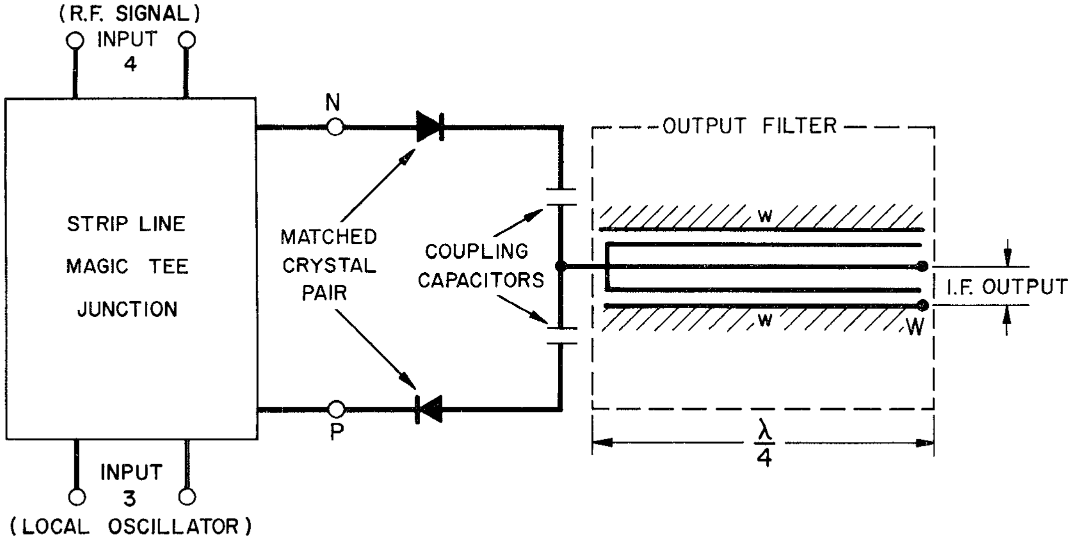

Fig. 6—Transmission line diagram of balanced mixer.

impedances are 225 ohms, 150 ohms, and 100 ohms respectively. With $f_4 = 450$ mc and $\epsilon_r = 2.5$ (Teflon fiberglass),

$$\lambda_4/4\sqrt{\epsilon_r} = 4.125 \text{ inches.}$$

Thus far in the design process no specification had been made for the characteristic impedances $Z_{0_{ca}}$, $Z_{0_{de}}$, $Z_{0_{bc}}$, $Z_{0_{aw}}$, and $Z_{0_{cw}}$. Wideband impedance matching, however, dictates that these values be as high as possible. Other factors restrict this choice, since in each of these transmission lines one or both conductors enter into the determination of other characteristic impedances. Because the final values of $Z_{0_{cw}}$, $Z_{0_{de}}$, $Z_{0_{aw}}$, and $Z_{0_{cw}}$ result from compromises between various design aspects including the physical configuration of the several conductors, it is well to discuss here the physical construction.

The dielectric sheets employed were of a standard thickness; consequently the width of conductor *b* is established by the required value of $Z_{0_{ba}}$. Since $Z_{0_{bc}} = Z_{0_{ba}}$ (225 ohms in the present case), the shunt reactance across nodes $M-Q$ due to the shorted length l_1 of transmission line $b-c$, is high at the frequency, f_4. Thus this shunting reactance will have a small effect.

To avoid coupling between inputs 3 and 4, the leakage field from conductor *b* must be kept to a minimum. Zero leakage would require conductors *c* and *a* to be infinitely wide; however, for widths greater than twice that of *b* the leakage is negligibly small.[4] The width of *b* is approximately 1/64 inch; thus making *a* and *c* 1/16 inch wide insures adequate shielding of *b*. Widths *a* and *c* are fixed in this way, and the spacing to the ground planes is selected to give a high value of $Z_{0_{cw}}$ and $Z_{0_{aw}}$. With a 1 inch spacing between the ground planes this value becomes 210 ohms.

The quarter wavelength short-circuited sections of lines *cw* and *aw* appear directly in parallel with the crystal diodes for an input at 3. These lines have a high characteristic impedance. This fact, together with the fact that no other shunt reactances occur and the required impedance transformation is only 2.2:1, yields a very broad-band impedance match for input 3.

Returning to considerations for input 4: $Z_{0_{ac}}$ and $Z_{0_{de}}$ remain to be determined. $Z_{0_{ac}}$ actually has already been fixed at 80 ohms by the standard dielectric thickness and the previously selected widths of *a* and *c*. The widths of conductors *d* and *e* were chosen as small as practically possible to yield a value of $Z_{0_{de}} \approx 220$ ohms. Assuming

$$l_3 = l_1 = \frac{\lambda_4}{4\sqrt{\epsilon_r}},$$

then at the *NPW* plane, for input 4, the total load resistance of 440 ohms is shunted by a reactance

$$57 \tan \frac{\pi}{2} \frac{\lambda_4}{\lambda}.$$

From this it appears that the maximum bandwidth for a vswr of 2:1 is 25 per cent. Clearly the useful operating bandwidth of input 4 will be considerably more limited than that of input 3. The ground plane spacings to conductors *d* and *e* was reduced to maintain $Z_{0_{dw}}$ and $Z_{0_{ew}}$ at the values required for the proper impedance transformation from input 3.

The values of the various transmission line impedances employed in the balanced mixer are listed in Fig. 7. At the crystal output, the transmission line band reject filter is equivalent to the choke type conventionally employed in coaxial lines.

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[4] F. Assadourian and E. Rimai, "Simplified theory of microstrip transmission systems," PROC. IRE, vol. 40, pp. 1651–1657; December, 1952.

$Z_{0_3} = Z_{0_4} = 50$ ohms

$Z_{0_{aw}} = Z_{0_{cw}} = 210$ ohms

$Z_{0_{ba}} = Z_{0_{bc}} = 225$ ohms

$Z_{0_{dw}} = Z_{0_{ew}} = 149$ ohms

$Z_{0_{ac}} = 80$ ohms

$Z_{0_{de}} = 220$ ohms

Three stage quarter-wave section transformer 50 ohms–440 ohms.

Characteristic impedance of quarter-wave sections 100 ohms, 150 ohms and 225 ohms.

Fig. 7—Characteristic impedances in hybrid junction mixer.

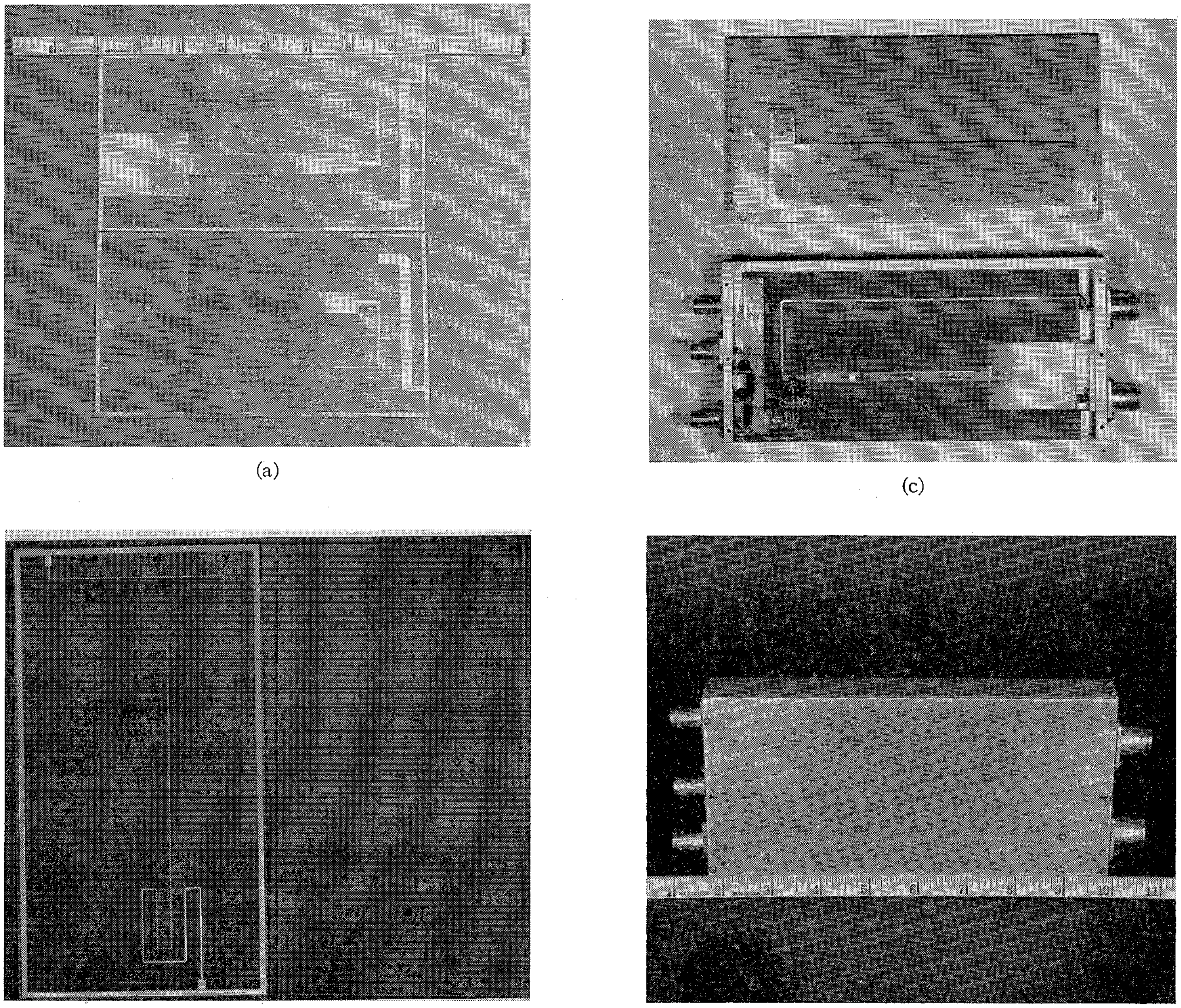

(a) (c) (b) (d)

Fig. 8—(a) Printed board components on the sides facing the ground planes. (b) Sides facing each other. (c) Mixer with one of the ground planes removed. (d) Mixer completely assembled.

The construction details are shown in Fig. 8(a) to 8(d). The folded conductor seen in Fig. 8(b) is conductor *b*. Folded construction is employed to conserve space. For the particular dimensions and spacing chosen, the folded conductor differs very little from the equivalent straight conductor in the uhf region. Conductors *a* and *c* were correspondingly enlarged to form the necessary outer conductor.

Type IN263 crystal diodes were employed because of their convenient construction and uniformity of electrical characteristics.

MEASURED DATA ON STRIP LINE HYBRID JUNCTION MIXER

In Fig. 9 (opposite) is plotted the input vswr of the two inputs when l_2 was adjusted to tune input 3 to a center frequency of 560 mc, l_1 was equal to l_2, but l_3 was set to tune input 4 at a center frequency of 350 mc. Since the quarter-wave transformers for input 3 and 4 were designed for 390 mc and 450 mc respectively, there was, in this case, a considerable displacement from the optimum operating frequencies. As a consequence the bandwidths and match at the center frequencies were some-

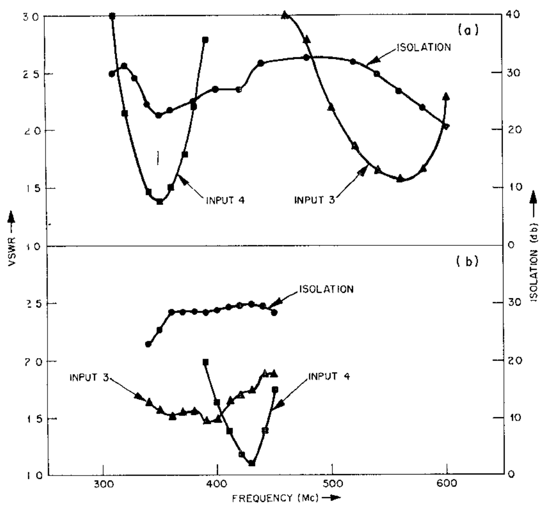
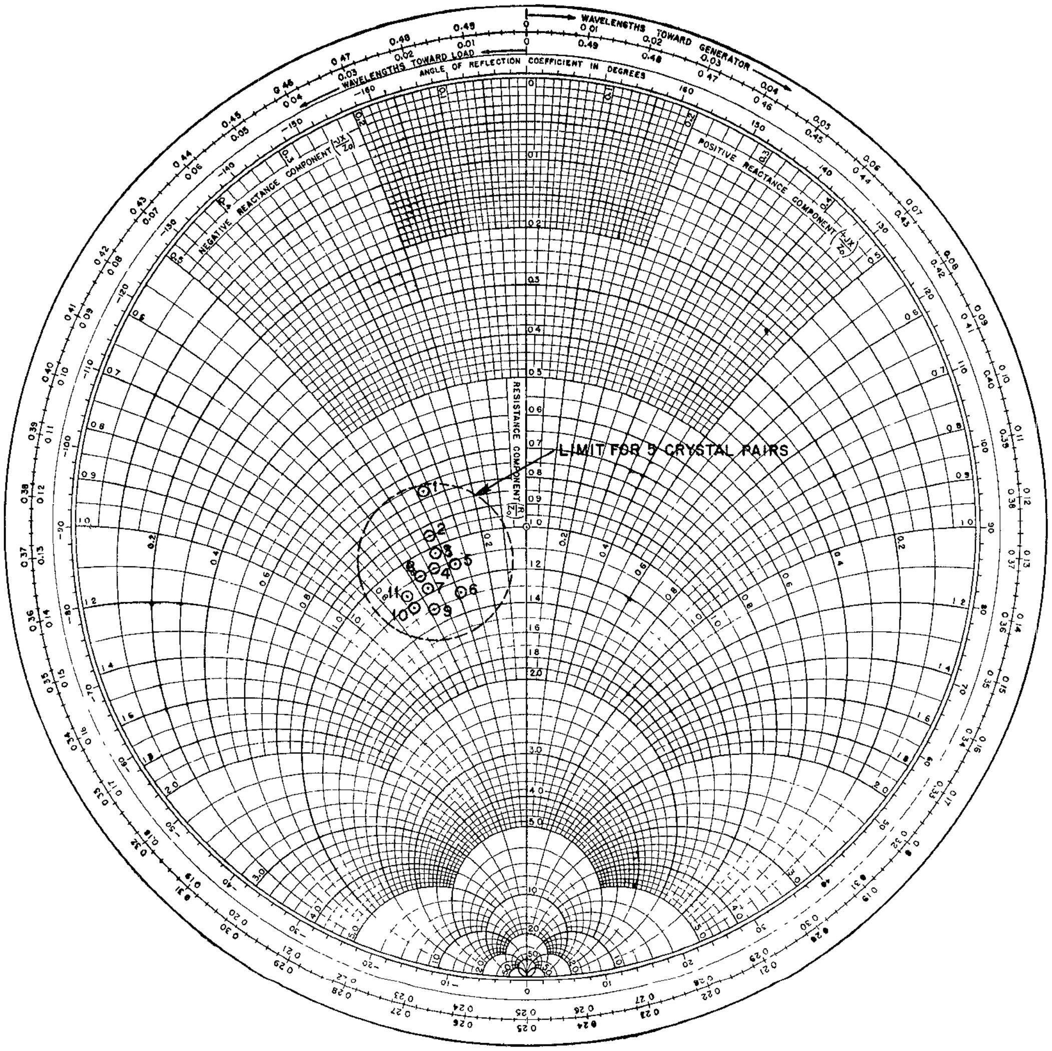


Fig. 9—Strip line magic tee mixer 1N263 crystals $I_x = 0.6$ ma. Input vswr and isolation vs frequency two sets [(a) and (b)] of l_1, l_2, l_3.

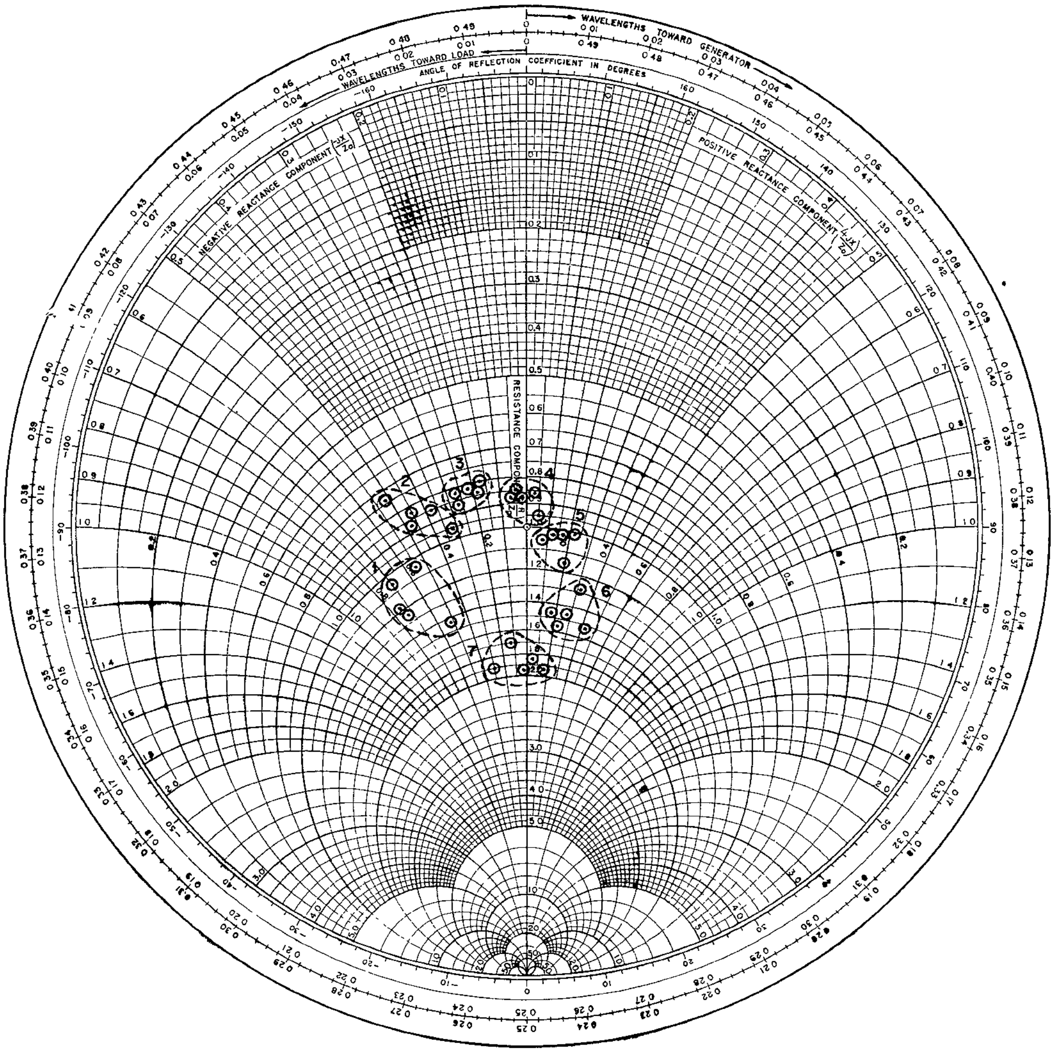
what impaired, but are still adequate for many applications. For a vswr $\leq 2.0{:}1.0$ input 3 had a 15.2 per cent bandwidth and input 4 a 14.8 per cent bandwidth. Over the entire range from 300 mc to 600 mc the isolation between the two inputs exceeds 20 db. Fig. 9(b) gives the same data for the case where the inputs were tuned to frequencies corresponding to those of their quarter-wave impedance transformers. In this case for vswr 2.0:1.0, bandwidths were 3–36 per cent, 4–16.3 per cent. These results were very closely repeated by other crystals as shown in Fig. 10 where the input 3 impedance is given as a function of frequency for five different crystal pairs. For the data shown in Figs. 9(b) through 12, l_1, l_2, and l_3 were kept constant. Fig. 11 (next page) gives the impedance spread for five crystal pairs as seen from input 4. Fig. 12 shows the measured range of isolation for five crystal pairs for frequencies 340 mc through 460 mc.



Point Frequency (mc)

1—340
2—350
3—360
4—370
5—390
6—400
7—410
8—420
9—430
10—440
11—450

Fig. 10—Impedance vs frequency, input 3 typical pair of 1N263 crystals $I_x = 0.6$ ma.



Group Frequency (mc)
1—390
2—400
3—410
4—420
5—430
6—440
7—450

Fig. 11—Impedance vs frequency, input 4 five pairs of 1N263 crystals $I_x = 0.6$ ma.

CONCLUSION

A strip line hybrid junction has been devised which maintains a relatively high degree of isolation between two inputs over a two to one frequency range. The inputs are capable of being independently tunable anywhere in these frequency ranges. Since the configuration is easily reproducible by printed circuit techniques, economy and space reduction are also available for the magic tee as well as hybrid rings now commonly used in strip line work. It would appear that this form of hybrid junction would be particularly well suited for applications in the uhf and vhf regions.

ACKNOWLEDGMENT

The author is indebted to E. Maxwell and C. E. Chase for helpful comments, and to P. S. Ambeau and D. F. Wiggin for assistance with the experimental work.

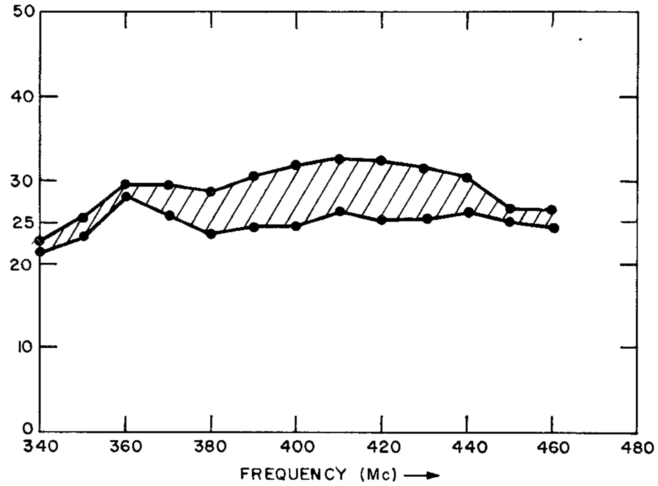


Fig. 12—Strip line magic tee mixer max and min isolation of five (1N263) crystal pairs.